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Period: 1

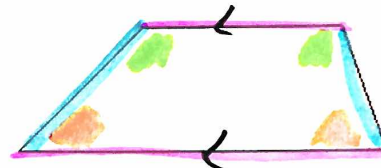
Trapezoids and Kites

Lesson Objectives | WBAT prove a quad. is a trapezoid in the coordinate plane; apply thms about trapezoids

Trapezoid (DEF): A trapezoid is a quadrilateral with exactly one pair of parallel sides.

→ The parallel sides are called the bases.

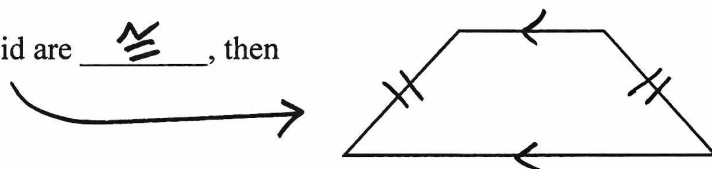
→ The non-parallel sides are called the legs.



Base Angles: The base angles of a trapezoid are two consecutive angles whose common side is a base.

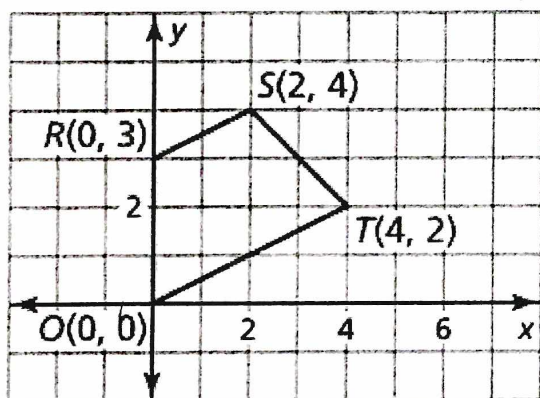
→ A trapezoid always has 2 pairs of base angles

Isosceles Trapezoid: If the legs of the trapezoid are $\cong$, then the trapezoid is isosceles



Example:

1. Show that ORST is a trapezoid. Then decide whether it is an isosceles trapezoid.



$$\text{slope } \overline{RS} = \frac{1}{2} \quad \text{slope } \overline{OT} = \frac{2}{4} = \frac{1}{2}$$

$$\text{slope } \overline{OR} = \text{undef.} \quad \text{slope } \overline{ST} = -1$$

Since $\overline{RS}$, $\overline{OT}$ have same slope, $\overline{RS} \parallel \overline{OT}$. Since $\overline{OR}$, $\overline{ST}$ do not have same slope, $\overline{OR}$ is not $\parallel$ to $\overline{ST}$.

Since ORST has exactly one pair of $\parallel$ sides, ORST is a trapezoid.

$$\text{isos??} \quad OR = 3$$

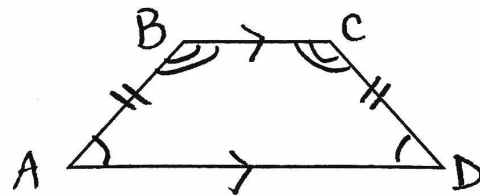
$$ST = \sqrt{(2-4)^2 + (4-2)^2} = 2\sqrt{2}$$

Since $\overline{OR} \neq \overline{ST}$, OR is not $\cong$ to $\overline{ST}$. therefore, ORST is not isos.

Isosceles Trapezoid Base Angles Theorem

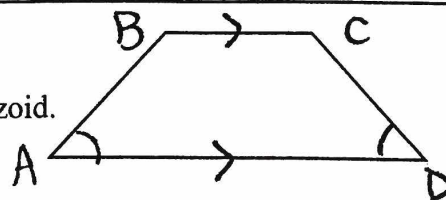
If a trapezoid is isosceles, then each pair of base angles is congruent.

$$\angle A \cong \angle D, \angle B \cong \angle C$$



Isosceles Trapezoid Base Angles Converse

If a trapezoid has a pair of congruent base angles, then it is an isosceles trapezoid.



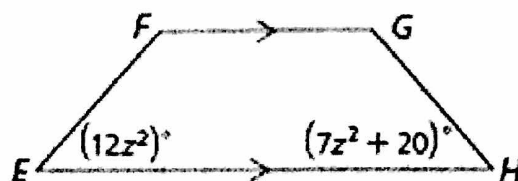
Example:

2. Find the value of z so that trapezoid EFGH is isosceles.

$$12z^2 = 7z^2 + 20$$

$$5z^2 = 20$$

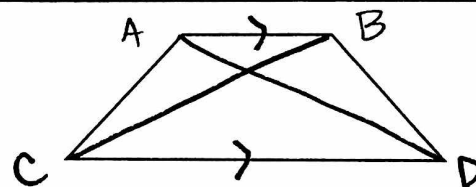
$$z^2 = 4 \rightarrow \boxed{z = \pm 2}$$



Isosceles Trapezoid Diagonals Theorem

A trapezoid is isosceles if and only if its diagonals are congruent.

$$\overline{BC} \cong \overline{AD}$$

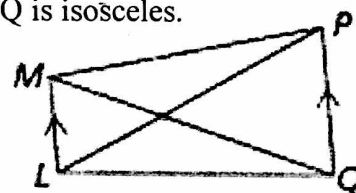


Example:

3. $MQ = 7y - 6$ and $LP = 4y + 12$. Find the value of y so that trapezoid LMPQ is isosceles.

$$7y - 6 = 4y + 12$$

$$\boxed{y = 6}$$



REVIEW: A midsegment of a triangle is the segment that connects midpoints of 2 sides.

The midsegment of a trapezoid is the segment that connects midpoints of legs.

Trapezoid Midsegment Theorem

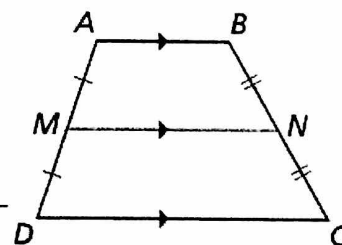
The midsegment of a trapezoid is...

1. // to each base

2. $\frac{1}{2}$ the length of the sum of the lengths of the bases

avg. of bases

$$MN = \frac{1}{2}(AB + CD)$$

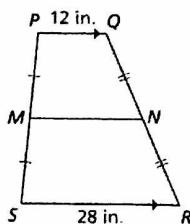


Example:

4. In the diagram, $\overline{MN}$ is the midsegment of trapezoid PQRS. Find MN.

$$MN = \frac{1}{2}(12 + 28)$$

$$\boxed{MN = 20}$$

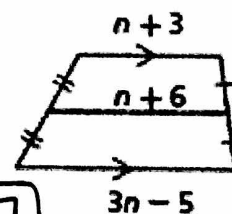


5. Find the value of n .

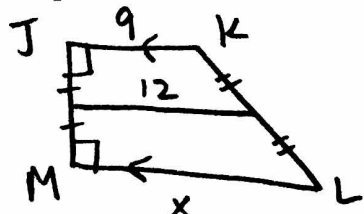
$$n + 6 = \frac{1}{2}(n + 3 + 3n - 5)$$

$$n + 6 = \frac{1}{2}(4n - 2)$$

$$n + 6 = 2n - 1 \rightarrow \boxed{n = 7}$$



6. In trapezoid JKLM, $\angle J$ and $\angle M$ are right angles, and $JK = 9$ cm. The length of midsegment $\overline{NP}$ of the trapezoid is 12 cm. Sketch the trapezoid and its midsegment, and find the length of $\overline{ML}$.



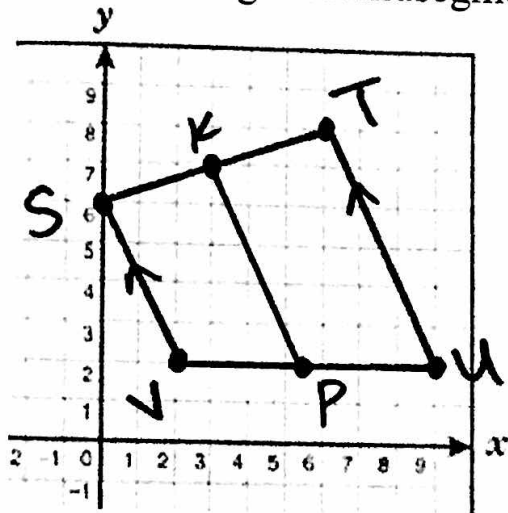
$$12 = \frac{1}{2}(9 + x)$$

$$24 = 9 + x$$

$$15 = x$$

$$\boxed{ML = 15}$$

7. Find the length of midsegment $\overline{YZ}$ in trapezoid STUV.



① midpts of each leg ($\overline{TS}$ and $\overline{UV}$)

$$P(5.5, 2) \quad K(3, 7)$$

$$PK = \sqrt{(3 - 5.5)^2 + (2 - 7)^2}$$

$$= \sqrt{(-2.5)^2 + (-5)^2}$$

$$= \sqrt{\left(-\frac{5}{2}\right)^2 + 25}$$

$$= \sqrt{\frac{25}{4} + \frac{100}{4}}$$

$$\begin{aligned} &= \sqrt{\frac{125}{4}} \\ &= \frac{\sqrt{125}}{\sqrt{4}} \\ &= \boxed{\frac{5\sqrt{5}}{2}} \end{aligned}$$